



Cryptography and Architectures for Computer Security

Exam Code: 095947 (old 090959), A.Y. 2014–2015, Semester: 2

Prof. G. Pelosi

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Name: Surname:

Student ID: Signature:

Time: 2h:30'. Use of textbooks, notes, phones or Internet connected devices is not allowed. Prior to turn in your paper, write your name on any additional sheet and sign it.

Question 1 [6 pts]

- What are the choices of RSA and DSA key lengths to provide a sufficient security to be employed with a 128-bit symmetric cipher? What are the ones for a 256-bit symmetric cipher?
- Discuss if and when re-encrypting a message with the same block cipher employing different keys improves the security of system.
- Discuss if re-encrypting a message with an RSA cryptosystem employing different public exponents and the same public modulus improves the security of system.

Question 2 [2 pts]

- To define a cryptographic hash function, assume the input message m to be divided into l blocks of length L bits: $m = \langle m_1, m_2, \dots, m_l \rangle$. Let $h(x) = m_1 \oplus m_2 \oplus \dots \oplus m_l$. Is $h(\cdot)$ a good cryptographic hash function?
- Describe the Merkle–Damgård design of hash functions and show how to realize a proper “keyed” hash function with it.

Question 3 [4 pts]

- Are there any differences in signing an integer value a representing an amount of money with RSA and signing $\text{SHA-2}(a)$?
- Consider the case where a user decides to employ his RSA secret decryption exponent d also as his static Diffie-Hellman secret exponent, to lower the stress imposed on the system random number generator.
 - Is it possible to violate the communication confidentiality whenever the DH-agreed key is used?
 - Is it possible to violate message authentication provided via RSA signatures for some messages? if yes, how?

Question 4 [6 pts]

Consider the multiplicative cyclic group $G=(\mathbb{Z}_{49}^*, \cdot)$:

- (a) Show the number of generators and the values of at least two of them, denoting them g and h , respectively. Discuss the existence of the following discrete logs and compute them, if possible.

$$\begin{aligned}x_0 &= \log_g^D 35 \\x_1 &= \log_g^D 25 \\x_2 &= \log_h^D 25 \\x_3 &= \log_h^D (25 \cdot g)\end{aligned}$$

- (b) Show the value of the generators of the two largest (proper) subgroups.
- (c) What are the criteria for selecting the parameters of a Diffie-Hellman key exchange protocol based on the algebra of cyclic groups?

Question 5 [6 pts]

Alice employs the Elliptic Curve Digital Signature standard (EC-DSA) protocol to sign her messages. Denoting as \mathcal{O} the point at infinity, assume the following public parameters of the scheme.

$$\mathbb{E}(\mathbb{F}_{31}) : \{x, y \in \mathbb{F}_{31} \text{ s.t. } y^2 = x^3 + 9x + 1\} \cup \{\mathcal{O}\}$$

$n=|\mathbb{E}(\mathbb{F}_{31})|=35$ (order of the curve), $P=(7, 2)$ (base point).

- (a) Compute the order of P , and $Q=-P$ justifying every passage.
- (b) Compute the signature of the message $m_1 \in \{0, 1\}^*$, that is: $(k_1, z_1) = \text{Sign}_{k_{priv}}^{\text{EC-DSA}}(m_1)$, assuming digest $e_1 = \text{SHA-1}(m_1) \equiv_{35} 12$, $r=8 \in \mathbb{Z}_n^*$ as random nonce extracted by the signature procedure, and $s=31$ as Alice's private key.
- (c) Alice sends another signed message as a pair $\langle m_2, (k_2, z_2) \rangle$.
The message $m_2 \in \{0, 1\}^*$ has digest $e_2 = \text{SHA-1}(m_2) \bmod n \equiv_{35} 18$, and the signature is $(k_2, z_2) = \text{Sign}_{k_{priv}}^{\text{EC-DSA}}(m_2) = (28, 32)$.
An adversary is able to collect the data transmitted with both messages and derive the Alice's secret key.
Explain the mistake made by Alice and show the computation made by the adversary to derive the value of the secret key s .

Hint: EC-DSA Signature Algorithm: $k_{priv} \leftarrow (s)$, $s \in \mathbb{Z}_n$ and $k_{pub} \leftarrow (n, P, [s]P)$

1. $r \xleftarrow{\text{Random}} \{1, \dots, n-1\}$, $\text{gcd}(r, n) = 1$
2. $[r]P = (x_1, y_1)$, $k \leftarrow x_1 \bmod n$. if $k = 0$ then go to step 1.
3. $e \leftarrow \text{SHA-1}(m)$
4. $z \leftarrow r^{-1}(e + s \cdot k) \bmod n$. if $z = 0$ then go to step 1.
5. $\text{Sign}_{k_{priv}}^{\text{EC-DSA}}(m) = (k, z)$

Question 6 [9 pts]

- (a) Apply the Pollard's ρ method¹ to factor the RSA public modulus $n = p \cdot q = 1517$
- (b) Given the private exponent $d=997 \bmod \varphi(n)$, compute the corresponding RSA public exponent employing the extended Euclidean algorithm
- (c) Apply the CRT algorithm to sign the plaintext message $m=42 \in \mathbb{Z}_n$
- (d) Assume to work into the Montgomery domain: $(\mathbb{Z}_N, +, \times)$. $N=27$. Compute the Montgomery multiplication $C=A \times B \bmod N$, where $A=19_{\text{decimal}}=103_4$ and $B=23_{\text{decimal}}=113_4$ are Radix-4 encoded values in the Montgomery domain. Show every step of the procedure, performing all the computations in Radix-4.

¹as a back-up strategy you can apply a trivial division method